A Love Letter to Optimization

Introduction

“Wait wait, hold on!!! Is there something wrong in the title and the cover picture? How does a picture of packing a luggage to the Bahamas (or whatever it is on your traveling list that you have been trying to tick-off) have anything to do with a mathematical concept like optimization? Is it clickbait?“. You might ask and for good reason. Well, I have a habit to explain difficult concepts in the most intuitive way possible (for better or worse really) so stay with me. However, if you want to dive deep into the mathematical behind it, I would suggest the following book:

So imagine you are facing a following task:

“You are packing for a trip to the beach and the following table shows a list of stuff that you could fit into your carry-on luggage (sorry, trying to save some money here!). Each stuff will have a volume (how big it is) and an utility (how useful is it). You must bring something to cover your foot, top-half, and bottom-half (duh!). Which item should you bring with you to maximize the utility? Your bag can only fit up to a volume of 8.”

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| --- | --- | --- |
| Item | Volume | Utility |
| Sandal | 2 | 4 |
| Oxford | 2 | 2 |
| Hawaiian shirt | 3 | 5 |
| Oxford Cloth Button Down | 3 | 3 |
| Grey Trousers | 4 | 2 |
| Beige Shorts | 2 | 5 |

Believe it or not, the task above is an optimization problem and it is commonly referred to as a “knapsack problem”. Let’s construct it in mathematical terms. First step, let’s construct the decision variable. Often times, the decision variable is right there in the question, i.e., which item should you bring. In other times, the decision variable is rather tricky and a poor choice of decision variable can let to a bad optimization model. That’s for another time. In this case, Let’s introduce representing whether or not to bring a pair of sandal:

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This sandal has other characteristic as well like volume and utility and we can denote it as , and . We can do the same thing as well for other items like , , etc... While writing this blog, I’m really tired having to type the item name again and again. What is there are 1000 item? Should I write it all out too? Thankfully, mathematician creates the term “Set” exactly for this. We have a Set of Item *I={1,2,3...6}* and each number represents an item (i.e. 1 stands for sanda -> 6 for beige shorts).

Second, let’s set up our objective. Again, this would rather be easy in this context since the answer is right in the question, i.e., maximizing utility. However, it’s not always that easy like what a government should do to maximize its citizen happiness? Lot’s of problem can prompts up like on what metric do you measure happiness? Is it GDP or the level of dopamine in one’s brain? For GDP, If there’s an economic policy that would bring happiness to the majority of the people but not to the remaining (like using one wealthy city revenue/tax to subsidize severals poorer one), should we still do it? I’m straying here, let’s hop back to our “knapsack” problem. If we bring a sandal (i.e. we would gain in utility or in other words, . If we don’t bring a sandal (i.e. , the product would be 0. We can do the same thing for other item and take the sum in mathematical term as follow:

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| --- | --- | --- |
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People love to use the parameter to represent the objective function value so if you have no preference on what you should call it, just use for peace of mind.

Third, let’s set up our constraints. Let’s tackle the knapsack capacity first



Objective: What are you trying to achieve (i.e. maximizing the Utility)?

Decision variable: a variable that would chiefly affect the objective function value (i.e. what item to bring)

Constraint: limitation on how far your variable can go (i.e. at least something to cover your foot, top-half, and bottom-half; the capacity of your carry-on)

A Solution: a combination of decision variables (i.e. an oxford shoe, OCBD, and Beige Short) t